did on the unidirectional plates. The poor performance of the group 1C plates might be attributable to the inability to fabricate these plates with the reoriented fibers conforming closely to the hole boundary; a more sophisticated fabrication procedure might improve these results significantly. The superiority of the group 2C plates over the group 2B plates indicates that the fiber reorientation concept has the potential for providing significant improvements in strength/weight ratios with little increase in fabrication costs. A comprehensive experimental and analytical study is currently being made to obtain details of the stress distribution around the hole. Future work should investigate other loading conditions and structures to which the fiber reorientation concept can be applied.

References

[1] "The Potentials of Composite Structures in the Design of Aircraft," Advisory Rept. 10, 1967, AGARD.

[2] Steg, L., "Recent Research on Composite Materials and Structures," presented at the Meeting of the Japan Society of Aeronautical and Space Sciences, April 1968.

[3] "Structural Design with Fibrous Composites," Publication MAB-236, National Academy of Sciences—National Academy of Engineering, Oct. 1968.

Capacity of Small-Car Transit Systems

J. Edward Anderson*

University of Minnesota, Minneapolis, Minn.

Nomenclature

- A = rate of deceleration of a transit car into a station
- C = capacity of a transit line in people per unit time
- g = acceleration of gravity
- H = distance between vehicles or headway
- L_c = length of a transit car
- ΔL = minimum tolerable spacing between cars in deceleration line
- n = maximum permissible total acceleration in a curve, g
- P_c = number of people per car
- R_c = horizontal radius of curvature at the center of a curve
- R_v = vertical radius of curvature at the center of a curve
- t_d = time required to clear a car from the deceleration line
- t_s = time required to stop a car from line speed
- Δt = minimum time interval between cars on the line
- V_L = line velocity, i.e., the velocity on straight portions of the track
- V_c = velocity at the center of a curve
- x_f = position of nose of car when it has stopped (see Fig. 2)
- x_s = position of nose of car when it begins to decelerate (see Fig. 2)

Introduction

THIS Note examines the capacity of small, computer-controlled cars running on exclusive right-of-ways. Several forms of these systems are under development in the United States. Reference 1 describes this concept, Ref. 2 compares various of these systems, and Ref. 3 describes one particular system in detail. By making the cars small, the waiting time for transportation can be reduced to a minimum, it becomes practical to load in one car riders bound only for a particular station so that nonstop service can be provided on a particular line, and the required weight of the support structure is minimized. By making the width of the car sufficient for a single seat only, the width of the right-of-way is minimized.

Received April 14, 1969; revision received May 26, 1969.

* Associate Professor of Mechanical Engineering. Associate Fellow AIAA.

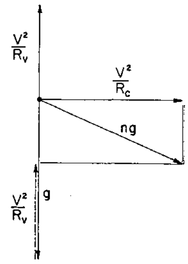

Fig. 1 Normal acceleration diagram for a vehicle in a curve.

The capacity of the whole system can obviously be increased by decreasing the distance between parallel lines or by adding lines in an overloaded right-of-way. The complete analysis of system capacity and consideration of the problems of control of a large number of small cars must be done by computer simulation. Such an analysis, and of course the system, is complicated considerably if each car is allowed to travel everywhere in the transit network. As suggested by Stelson,[4] however, the complexity of operation, initial development, and allowance for growth of the system can be considerably reduced if each line is an independent system, thus, only requiring the rider to transfer once, at most, in traveling from his origin to his destination. By proper design of the transfer stations, the inconvenience and discomfort of a transfer can be reduced far below that common today.

Capacity of a Given Line

In this analysis, we assume a small-car transit system in which each car is dispatched directly from its origin to a given destination and bypasses all intermediate stations at line speed. Problems connected with maintenance of line speed and spacing and the details of switching arrangements into and out of stations are not considered as they do not enter the calculations.

If the line is straight, its capacity C is simply

$$C = P_c V_L/(H + L_c) \tag{1}$$

If the line contains curves, the capacity is determined conservatively by replacing V_L in Eq. (1) by the maximum permissible velocity in the curve, V_c. This quantity is determined in a level banked curve from the formula

$$V_c^2/R_c = (n^2 - 1)^{1/2} g \tag{2}$$

In some circumstances, it may be desirable to make R_c as small as say 100 ft.[2] Then, if we take $n = 1.2$, $V_c = 31.6$ mph. As a line speed of 50–60 mph is usually appropriate, we see that sharp flat curves considerably reduce the system capacity.

Some improvement in capacity can be made and at the same time the car can be reliably and passively slowed down by causing the car to ascend into and descend out of the turn. At the center of the curve, the track would be raised enough to reduce V_L to V_c without mechanical braking, and would be designed to have negative curvature in the vertical plane so that as the car follows the track it would experience a downward centrifugal force tending to cancel the force of gravity. The result would be that with the same total acceleration on the passengers, the velocity in the curve would be higher than in a flat curve. This is illustrated in Fig. 1, from which we see that the total acceleration on the vehicle is given by

$$(ng)^2 = (g - V_c^2/R_v)^2 + (V_c^2/R_c)^2 \tag{3}$$

In this equation, we assume that n and R_c are fixed and that we want to choose the radius of curvature in the vertical direction R_v so that V_c is a maximum. This occurs when $V_c^2 = gR_v$ and hence when $R_v/R_c = n$. Thus

$$V_c^2 = ngR_c \tag{4}$$

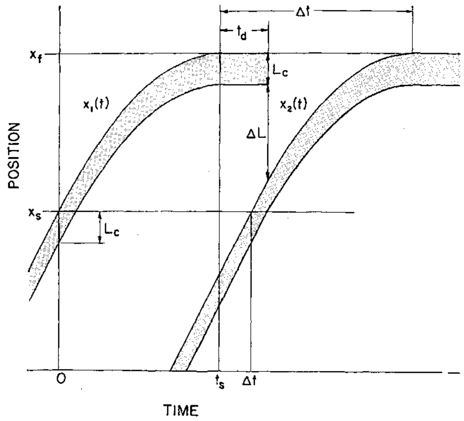

Fig. 2 Position-time diagram of two successive cars approaching a station.

We see that V_c is a maximum if the vertical centrifugal force at the center of the curve exactly cancels the force of gravity. This means that if the passenger is to feel no sideways force, the turn banks 90° at its center. A comparison of Eq. (4) and Eq. (2) shows that by using climbing turns the velocity in the turn can be increased by the ratio

$$(V_c)_{\text{climbing turn}}/(V_c)_{\text{flat turn}} = n^{1/2}/(n^2 - 1)^{1/4} \tag{5}$$

For example, for $n = 1.2$, this ratio is 1.34, so that V_c in the preceding example can be increased from 31.6 to 42.3 mph. The whole curve would of course have to be designed so that the acceleration experienced by the passenger changes very gradually from one g to the maximum and back, but this is a simple kinematical problem.

The advantages of the climbing turn are 1) that it can significantly increase the maximum capacity of the system, but, more important, 2) that it slows the vehicle to the required speed with 100% reliability without reliance on mechanical braking. Disadvantages are 1) that the requirement of 90° banking for maximum turning velocity would be felt to be disconcerting to passengers, and 2) that provisions would have to be made for the possibility that a car may pass through the turn at less than design speed. The first disadvantage can be easily overcome if the vehicle travels in a tube at least through the turn and if the contours of the tube are twisted so that they appear straight to a passenger riding at nominal speed. His only sensation in passing through the curve would then be that of passing over a moderate hill. Testing would of course be required to determine acceptance of this type of banked turn. The second disadvantage occurs only in emergency situations and could be countered by properly supporting the vehicle.

Capacity into a Station

Each car in the transit system carries with it a code corresponding to its designation station. As it approaches a station, the code is read and it is either allowed to pass through or it is switched out of line and then into a loading dock. This will permit parallel loading and dispatching and hence minimum station delay. The problem here is to determine the maximum rate at which cars can be allowed to enter a given station, i.e., the capacity into a station.

The problem is illustrated in Fig. 2. Cars approaching the station begin to decelerate when the front of the car of length L_c is at $x = x_s$. Here we assume uniform deceleration A, so that the stopping time t_s is

$$t_s = V_L/A \tag{6}$$

The car comes to a complete stop after traversing the distance $x_f - x_s$ given by

$$x_f - x_s = V_L^2/2A \tag{7}$$

After a delay of t_d sec, the car is clear of the deceleration line. The problem is then to determine the minimum time interval Δt between the noses of two successive cars so that the second car is no closer than ΔL to the first at the instant it is removed from the deceleration line. If $t = 0$ when the front of the first car is at x_s, the $x - t$ curve for the front of the first car in the interval $x_s \leq x_1 \leq x_f$ is

$$x_1(t) = -\tfrac{1}{2}At^2 + V_Lt + x_s \tag{8}$$

The $x - t$ curve for the second car is found by substituting $t - \Delta t$ for t. Then at $t = t_s + t_d$, we see from Fig. 2 that

$$x_f - x_s = L_c + \Delta L - \tfrac{1}{2}A(t_s + t_d - \Delta t)^2 + V_L(t_s + t_d - \Delta t) \tag{9}$$

By using Eqs. (6) and (7), Eq. (9) simplifies to

$$\Delta L + L_c = \tfrac{1}{2}A(\Delta t - t_d)^2 \tag{10}$$

But the capacity into a station is simply $C = P_c/\Delta t$. Using Eq. (10) this gives

$$C = \frac{P_c}{t_d + [2(\Delta L + L_c)/A]^{1/2}} \tag{11}$$

Because of the square root, the most important parameters in determining C are the number of people per car P_c and the station delay time t_d. If tandem seating is used, L_c is a function of P_c of the form $L_c = a + bP_c$ where a and b are constants. Then from Eq. (11), C increases with P_c but not in direct proportion. But as P_c increases, the average number of vacant seats per car increases and the flexibility of the system decreases. Thus, the optimum value of P_c can be determined only from an over-all analysis of the whole system including economic and social considerations. As an example, suppose $P_c = 6$, $L_c = 20$ ft, $\Delta L = 20$ ft, $A = 5$ ft/sec², $t_d = 5$ sec. Then $C = 2400$ people per hour or one person every 1.5 sec.

The minimum permissible time interval into a given station is the denominator of Eq. (11). For a given system, this is a fixed predetermined number and can be used in a central dispatching computer together with the travel times between stations to develop a matrix of go, no go signals to indicate when it is permissible to dispatch a car to a given station.

Conclusions

The capacity of a small-car transit system should be stated in terms of two parameters: 1) the total capacity of a given line and 2) the capacity into a station. The first parameter is determined by the speed in curves and can be increased by using climbing turns. The second, given by Eq. (11), is independent of the line speed.

References

[1] "Future Urban Transportation Systems, Final Reports I and II," March 1968, Stanford Research Institute.

[2] Rust, L. W., "Mass Transit Systems: A Report to the Metropolitan Transit Commission of the Twin Cities Area," 1967, North Star Research and Development Institute, Minneapolis, Minn.

[3] Berggren, L. E., "Uniflo Mass Transit System Concept," Rosemount Engineering Co., Minneapolis, Minn.

[4] Stelson, T. E., "Transportation—Steps to the Ultimate," *Proceedings of the Second International Conference on Urban Transportation*, Pittsburgh, Pa., April 17–19, 1967, pp. 48–74.